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Forced convection heat transfer from a flat plate with an unheated starting length

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NOMENCLATURE

c_p	specific heat
f	non-dimensional stream function, ψ/ν
k, L	thermal conductivity and unheated plate length, respectively
N_x, Pr	Nusselt number and Prandtl number, respectively
R_x, R_L	Reynolds numbers defined with respect to x and L, respectively, $= Ux/\nu$, UL/ν
t, t_0, t_w	temperature, free-stream temperature and wall temperature for $x > L$, respectively
T	non-dimensional temperature, $(t-t_0)/(t_w-t_0)$
u, v	velocity components in x- and y-directions, respectively
U	free-stream velocity
x, y, X, Y	Cartesian coordinates, equations (2) and (7).

Greek symbols

β, ϕ	non-dimensional parabolic coordinates, equation (8)
β_0	value of β on the wall, $(2UX/\nu)^{1/2}$
η, σ	non-dimensional parabolic coordinates
$\theta, \theta_1, \theta_2$	temperature functions defined in equations (11) and (15)
ν, ρ	kinematic viscosity and density, respectively
σ_0, σ_1	values of σ on the wall for arbitrary x and for $x = L$, respectively, $(2Ux/\nu)^{1/2}$, $(2UL/\nu)^{1/2}$
ψ	dimensional stream function.

Superscript

$'$	differentiation with respect to η or ϕ.

Subscripts

x, y, xx, yy	partial derivative with respect to x, y, xx, yy, respectively
$\beta, \eta, \eta\eta, \sigma, \sigma\sigma$	partial derivatives with respect to β, η, $\eta\eta$, σ, $\sigma\sigma$, respectively.

1. INTRODUCTION

THIS note is concerned with the study of forced convection heat transfer over a semi-infinite plate with an unheated starting length. Eckert and Drake [1] first proposed an integral relation, which appears to be more valid for large Reynolds number flows, as is evident from the agreement with the solution of the boundary-layer energy equation. Later, Chao and Cheema [2], and Drake and Riley [3] studied this problem while Wahed *et al.* [4] studied the mixed convection problem using the boundary-layer approximations, which are valid for large Reynolds number. Drake and Riley [3] have presented some heat transfer results (see their Figs. 3 and 4) for a range of Reynolds numbers 0.001–20 and for different starting length Reynolds numbers (0–20) and conclude that the heat transfer rate decreases with increasing starting length Reynolds number. It is well known that for low Reynolds number flows one has to solve the vorticity equation [5] and that the boundary layer approximation becomes true [6] for $R_x \geqslant 1000$. As such the results of ref. [3] are not expected to be appropriate for low Reynolds number flows. However, the good agreement with Eckert's relation can be attributed to the fact that the analysis was based on the ratio of x/L, whereby the magnitude of the Reynolds number does not appear. With this in view, we have studied the heat transfer behaviour for various values of the Reynolds number for flow over a semi-infinite plate for low starting length Reynolds numbers. The vorticity equation rather than the boundary-layer equation has been considered in this study.

2. GOVERNING EQUATIONS

For the flow geometry shown on the inset of Fig. 1, the two velocity components in the x- and y-directions are u and v, respectively. The free-stream velocity and the temperature are denoted by U and t_0, respectively. The plate is heated for $x > L$. For $x < L$ the plate temperature is that of the free-stream. In the present analysis, the upstream diffusion of heat in the region $x < L$ will be neglected. Also, it will be assumed that the Reynolds number based on the unheated starting length is low.

In order to solve the vorticity and energy equations we use the parabolic coordinates, which are optimal [7]. Consequently the vorticity equations [6] can be expressed as

$$F'''' + F'''[F - 4\eta(\sigma^2+\eta^2)^{-1}] + (\sigma^2+\eta^2)^{-1}[(\sigma^2-\eta^2)F' - 2\eta F]F'' = 0 \quad (1)$$

where

$$x + \mathrm{i}\,y = \nu(\sigma + \mathrm{i}\,\eta)^2/2U. \quad (2)$$

Here primes denote derivatives with respect to η. (In ref. [6], equation (1) has been expressed in terms of σ_0, the value of σ on the wall. In order to solve equation (1) at any prescribed streamwise location σ will be replaced by σ_0.)

The boundary conditions are

$$\left.\begin{aligned} F = F' = 0 \quad &\text{at} \quad \eta = 0 \\ F \sim \eta, \quad F' \to 1 \quad &\text{as} \quad \eta \to \infty. \end{aligned}\right\} \quad (3)$$

The two velocity components are given by

$$\left.\begin{aligned} u &= [\eta F + \sigma^2 F']U/(\sigma^2+\eta^2) \\ v &= [\sigma\eta F' - \sigma F]U/(\sigma^2+\eta^2) \end{aligned}\right\} \quad (4a)$$

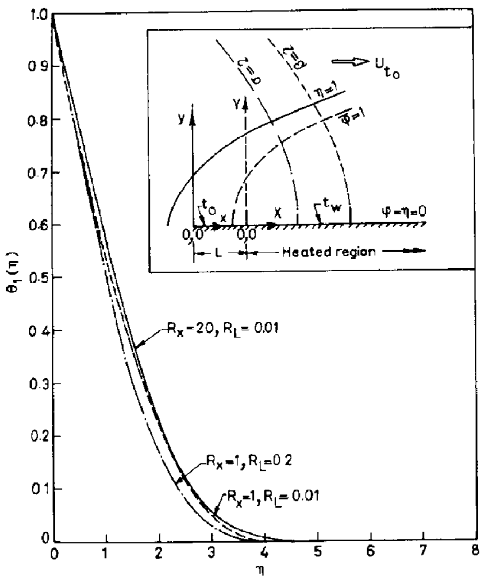

FIG. 1. Temperature profiles (θ_1) for different values of the local Reynolds number (R_x).

where the non-dimensional stream function is given by

$$f = \sigma[F(\eta)+(\sigma-\sigma_0)F_1(\eta)\ldots]. \quad (4b)$$

The energy equation is given by

$$uT_x+vT_y = (k/\rho c_p)(T_{xx}+T_{yy}). \quad (5)$$

The boundary conditions for equation (5) are

$$T = 1 \text{ at } y = 0; \quad T \to 0 \text{ as } y \to \infty, \quad x > L$$
$$T = 0 \text{ for all } y, \quad x < L. \quad (6)$$

To solve the energy equation (5), we define X and Y such that

$$X = x-L; \quad Y = y \quad (7)$$

and

$$X+\mathrm{i}\,Y = v(\beta+\mathrm{i}\,\phi)^2/2U. \quad (8)$$

Using equations (7) and (8), the energy equation (5) becomes

$$u[\beta T_\beta-\phi T_\phi]+v[\phi T_\beta+\beta T_\phi] = Pr^{-1}[T_{\beta\beta}+T_{\phi\phi}]U. \quad (9)$$

The corresponding boundary conditions are

$$T = 1 \text{ when } \phi = 0; \quad T \to 0 \quad \text{as} \quad \phi \to \infty. \quad (10)$$

We expand T about any arbitrary point ($\beta_0 = (2R_X)^{1/2}$) on the surface ($Y = 0$) as

$$T = [\theta(\phi)+(\beta-\beta_0)\theta_2(\phi)\ldots]. \quad (11)$$

Using equation (11) and retaining only the first term we obtain from equation (9)

$$-u\phi\theta'+v\beta\theta' = Pr^{-1}\,U\theta'' \quad (12)$$

where primes denote derivative with respect to ϕ.

Since u and v are prescribed in σ and η, we make use of the relations

$$Y = y; \quad y = v\sigma\eta/U; \quad Y = v\beta\phi/U \quad (13)$$

to integrate equation (12) in η only. From equation (13) we have $\phi = \eta(\sigma/\beta)$, so that equation (12) is expressed as

$$-\eta u(\sigma/\beta)^2\theta_1'+v\sigma\theta_1' = Pr^{-1}\,\theta_1''U. \quad (14)$$

Since u and v are known from equations (2) and (4) for any prescribed streamwise location (σ_0) and β_0 is related to σ_0 by $\beta_0 = \sigma_0(1-R_L/R_X)^{1/2}$, we express equation (14) as

$$Pr^{-1}(\sigma_0^2+\eta^2)\theta_1''+[\eta(1-R_L/R_X)^{-1} \times(\eta F+\sigma_0^2F')-\sigma_0^2(\eta F'-F)]\theta_1' = 0. \quad (15)$$

The boundary conditions are

$$\theta_1 = 1 \text{ at } \eta = 0; \quad \theta_1 \to 0 \text{ as } \eta \to \infty. \quad (16)$$

It may be noted that, here we have considered the case when $R_x/R_L \gg 1$ (say $R_x/R_L \geqslant 5$). When $R_L/R_x = 0$, equation (15) reduces to

$$Pr^{-1}\,\theta_1''+f\theta_1' = 0 \quad (17)$$

which is the same as that of ref. [6] (for forced flow) and ref. [8]. Also, equation (15) becomes singular when $R_L/R_x \to 1$ and an analysis similar to that of Messiter and Liñán [9] can be used to obtain the solution. However, this case is not presented here.

3. RESULTS AND DISCUSSION

Equations (2) and (15) satisfying the boundary conditions (3) and (16) were solved for $Pr = 0.7$ and various values of the local Reynolds number (R_x = 1–100), for two values of the starting length Reynolds number (R_L = 0.01; 0.2).

Non-dimensional temperature profiles are shown in Fig. 1. For a given R_L, as expected, the thermal boundary-layer growth is observed. Also, for a given R_x, we expect a thinner thermal boundary-layer with increasing R_L, which is also evident from the temperature profiles.

The present results differ from those of Drake and Riley [3] in the sense that we observe an opposite trend in the heat transfer rate. The results of Wahed *et al.* [4] show that in mixed convection flows a higher heat transfer rate is obtained with higher starting length Reynolds number, a trend qualitatively similar to the present results.

In order to obtain a qualitative comparison with Eckert's results, the Nusselt numbers given by [3]

$$N_x/R_x^{1/2} = 0.332Pr^{1/3}/[1-(R_L/R_x)^{3/4}]^{1/3} \quad (18)$$

were calculated for $R_L = 0.01$ and 0.2 and for R_x = 1–100. Here also, we observed a higher heat transfer rate, at a given streamwise location, with increasing R_L. This shows that the trend observed in the heat transfer rate with increasing R_L (Fig. 2) is correct. Further, in order to compare our heat-transfer result for boundary-layer flow with that of Sparrow and Minkowycz [10], we have solved equation (17) under conditions (16). The wall temperature gradient is found to be 0.4139 for $Pr = 0.7$, which is identical to that of ref. [10].

4. CONCLUSIONS

To sum up, heat transfer has been studied for flow over a semi-infinite plate for low unheated starting length Reynolds

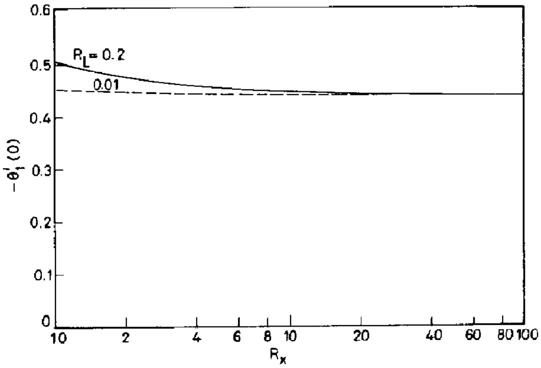

FIG. 2. Variation of the local wall heat transfer gradient with R_x for different values of R_L.

numbers using the vorticity equation and the results show that a higher heat transfer rate is obtained with an increase in the starting length Reynolds number.

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Heat transfer through a vertical enclosure with convective boundary conditions

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1. INTRODUCTION

IN THIS note we discuss the heat transfer through a vertical enclosure with convective boundary conditions. In a previous article we have reviewed some of the history of this problem and considered its relevance to double pane windows [1]. Elsewhere we have discussed the ramifications of replacing the air in the enclosed space by other fluids [2], and how the flow changes when the cavity is an enclosed annulus rather than one of Cartesian geometry [3].

In ref. [1] we took the sidewalls to be isothermal and considered the top and the bottom to be insulated. In this article we report numerical calculations for a more realistic model of a double pane window by taking into account the thermal interaction of the sidewalls with the surroundings. We do this by specifying some reasonable values for the outside heat transfer coefficients.

The aim of this study is to see how the sidewall temperatures vary with height, and how this variation influences the convection in the air gap and heat transfer through it. An important practical outcome of the calculations is that one can estimate the error made if the panes are assumed to be isothermal. Although we have neglected the vertical conduction of heat in the panes, this does not invalidate the analysis, because vertical conduction acts to smooth out temperature variations. As a result the error estimate is a conservative one and provides a bound for what can be expected in practice.

Our calculations are for a cavity with an aspect ratio equal to 20. This is sufficiently high for the flow to undergo a transition to a multicellular convective pattern as the Grashof number is increased to a value of the order of 10 000. It is still, however, too small to be relevant to tall windows. The results in ref. [1] give the optimum gap width for tall windows, thus we are excused for considering them further in this note. We also discuss in Section 3 the qualitative differences which are manifest when a constant wall heat flux is imposed on one wall, while the other is held at a fixed temperature.

2. ANALYSIS

Consider a rectangular region of aspect ratio $A = H/L$ in which H is the height and L is the width. The depth is sufficiently large for the flow to be two-dimensional. The LHS is exposed to an ambient at temperature T_h via a heat transfer coefficient h_l and the RHS interacts with a colder ambient at temperature T_c, with a heat transfer coefficient h_r between them. The temperature differences are assumed to be sufficiently small that the Boussinesq approximation is valid.

The equations and boundary conditions governing convection in this space are given in ref. [2] and are not repeated here, except for the sidewall thermal boundary conditions. These are

$$\frac{\partial T}{\partial x} + Nu_{lL}(1-T) = 0, \quad x = 0 \tag{1}$$

and

$$\frac{\partial T}{\partial x} + Nu_{rL}\,T = 0, \quad x = 1. \tag{2}$$

In equations (1) and (2) the two Nusselt numbers are $Nu_{lL} = h_l L/k$ and $Nu_{rL} = h_r L/k$. These are related to Nusselt